Review of anisotropic flow correlations in ultrarelativistic heavy-ion collisions

Zhou, You

Published in:
Advances in High Energy Physics

DOI:
10.1155/2016/9365637

Publication date:
2016

Document version
Publisher's PDF, also known as Version of record

Citation for published version (APA):
Zhou, Y. (2016). Review of anisotropic flow correlations in ultrarelativistic heavy-ion collisions. *Advances in High Energy Physics*, *2016*, [9365637]. https://doi.org/10.1155/2016/9365637

Hindawi Publishing Corporation
Advances in High Energy Physics
Volume 2016, Article ID 9365637, 13 pages
http://dx.doi.org/10.1155/2016/9365637

Review Article

Review of Anisotropic Flow Correlations in Ultrarelativistic Heavy-Ion Collisions

You Zhou

Niels Bohr Institute, University of Copenhagen, Blegdamsvej 17, 2100 Copenhagen, Denmark

Correspondence should be addressed to You Zhou; you.zhou@cern.ch

Received 27 May 2016; Accepted 14 July 2016

Academic Editor: Md. Nasim

Anisotropic flow phenomena are a key probe of the existence of Quark-Gluon Plasma. Several new observables associated with correlations between anisotropic flow harmonics are developed, which are expected to be sensitive to the initial fluctuations and transport properties of the created matter in heavy-ion collisions. I review recent developments of correlations of anisotropic flow harmonics. The experimental measurements, together with the comparisons to theoretical model calculations, open up new opportunities of exploring novel QCD dynamics in heavy-ion collisions.

1. Introduction

One of the fundamental questions in the phenomenology of Quantum Chromo Dynamics (QCD) is, what are the properties of matter at extreme densities and temperatures where quarks and gluons are in a new state of matter, the so-called Quark-Gluon Plasma (QGP)? [1, 2]. Collisions of high-energy heavy ions, at the Brookhaven Relativistic Heavy Ion Collider (RHIC) and the CERN Large Hadron Collider (LHC), allow us to create and study the properties of the QGP matter in the laboratory. This matter expands under large pressure gradients, which transfer the inhomogeneous initial conditions into azimuthal anisotropy of produced particles in momentum space. This anisotropy of produced particles is one of the probes of the properties of the QGP [3, 4]. It can be characterized by an expansion of the single-particle azimuthal distribution $P(\varphi)$:

$$P(\varphi) = \frac{1}{2\pi} \sum_{n=-\infty}^{+\infty} \overrightarrow{V}_n e^{-in\varphi}, \quad (1)$$

where φ is the azimuthal angle of emitted particles, $\overrightarrow{V}_n$ is the nth order flow vector defined as $\overrightarrow{V}_n = v_n e^{in\Psi_n}$, its magnitude v_n is the nth order anisotropic flow harmonic, and its orientation is symmetry plane (participant plane) angle Ψ_n. Alternatively, this anisotropy can be generally given by the joint probability density function (PDF) in terms of v_n and Ψ_n as

$$P(v_m, v_n, \ldots, \Psi_m, \Psi_n, \ldots) = \frac{1}{N_{\text{event}}} \frac{dN_{\text{event}}}{v_m v_n \cdots dv_m dv_n \cdots d\Psi_m d\Psi_n \cdots}. \quad (2)$$

In the last decade, the experimental measurements of anisotropic flow v_n [5–55], combined with theoretical advances from calculations made in a variety of frameworks [56–62], have led to broad and deep knowledge of initial conditions and properties of the created hot/dense QCD matter. In particular, the precision anisotropic flow measurements based on the huge data collected at the LHC experiments and the successful description from hydrodynamic calculations demonstrate that the QGP created in heavy-ion collisions behaves like a strongly coupled liquid with a very small specific shear viscosity η/s [63–68], which is close to a quantum limit $1/4\pi$ [69].

It has been investigated into great details of event-by-event fluctuations of single flow harmonic. Based on the measurements of higher-order cumulants of anisotropic flow [43, 48, 51, 74, 75] and the event-by-event v_n distributions [40], it was realized that the newly proposed Elliptic-Power function [76–78] gives the best description of underlying

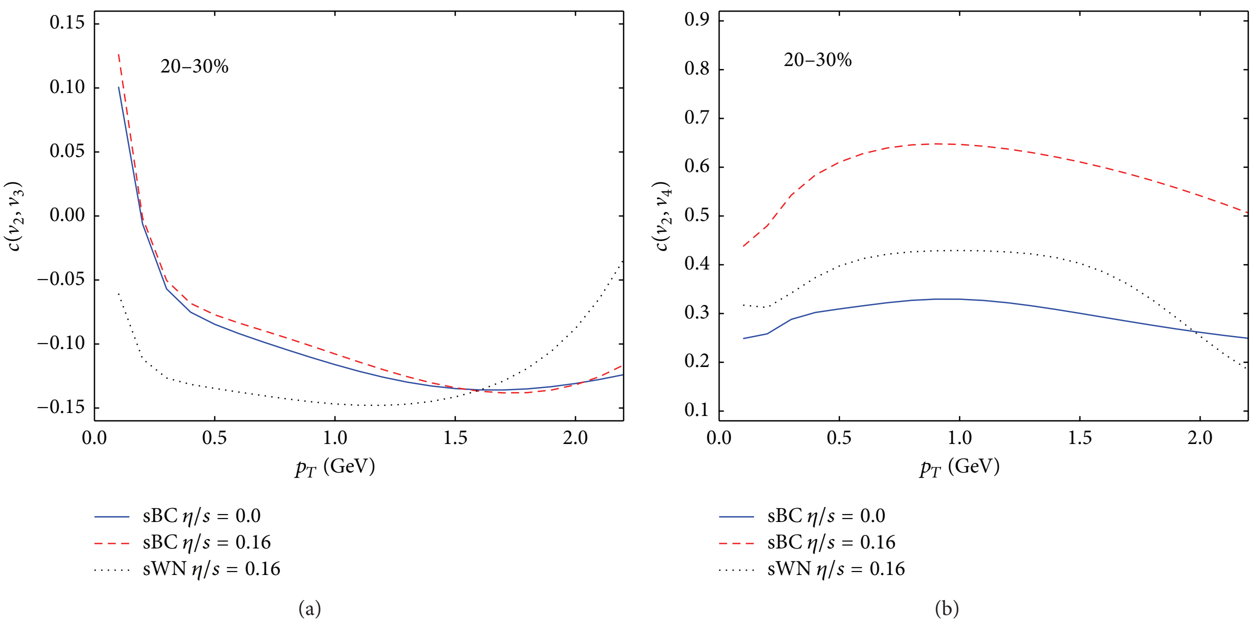

FIGURE 1: p_T dependence of $c(v_2, v_3)$ (a) and $c(v_2, v_4)$ (b) in centrality 20–30% in Pb–Pb collisions at $\sqrt{s_{\mathrm{NN}}} = 2.76$ TeV. Figures are taken from [83].

PDF of single harmonic v_n distributions [72, 79, 80]. On the other hand, it has been known for a while that both the flow harmonic (magnitude) v_n and its symmetry plane (orientation) Ψ_n of the flow vector $\overrightarrow{V_n}$ fluctuate event-by-event [81–83], but only recently p_T and η dependent flow angle (Ψ_n) and magnitude (v_n) were predicted by hydrodynamic calculations [84, 85]. Many indications were quickly obtained in experiments by looking at the deviations from unity of $v_n[2]/v_n\{2\}$ [86] and factorization ratio r_n [52, 55, 86]. These measurements were nicely predicted or reproduced by hydrodynamic calculations and are found to be sensitive to the initial state density fluctuations and/or the shear viscosity of the expanding fireball medium [84, 85, 87]. Most of these above-mentioned studies are focused on the fluctuations of single flow harmonics and their corresponding symmetry planes, as a function of collisions centrality, transverse momentum p_T, and pseudorapidity η. Results of correlations between symmetry planes [28, 41] reveal a new type of correlations between different order flow vectors, which was investigated in the observable of v_{2n/Ψ_n} before [88–90]. In particular, some of the symmetry planes correlations show quite different centrality dependence from the initial state and final state, and this characteristic sign change during system evolution is correctly reproduced by theoretical calculations [62, 82, 91], thus confirming the validity of hydrodynamic framework in heavy-ion collisions and further yielding valuable additional insights into the fluctuating initial conditions and hydrodynamic response [62, 82, 92].

In addition to all these observables, the (anti)correlations between anisotropic flow harmonics v_m and v_n are found to be extremely interesting [45, 62, 70, 71, 93]. A completely new set of information on the joint probability density function (PDF) can be obtained from the rich correlation pattern observed in experiments. On the other hand, no existent theoretical calculations [62, 70, 71, 93] could provide quantitative descriptions of data [36]. Thus, it is crucial to investigate in depth the relationship between different flow harmonics: whether they are correlated, anticorrelated, or not correlated from both experimental and theoretical points of view.

2. Correlations of v_n and v_m Fluctuations

It is found recently that the relationship between different order flow harmonics can be used to probe the initial state conditions and the hydrodynamic response of the QGP [36, 71, 93–95]. In order to better understand the event-by-event $P(\varphi)$ distribution, it is critical to investigate the relationship between v_m and v_n. Considering the naive ellipsoidal shape of the overlap region in noncentral heavy-ion collisions generating nonvanishing even flow harmonics v_{2n}, the correlations between the even flow harmonics are expected. However, it is not straightforward to use geometrical argument to explain the relationship between even flow harmonics for central collisions, where all the harmonics are driven by fluctuations instead of geometry, and to explain the relationship between even and odd flow harmonics for central and noncentral collisions [80]. A linear correlation function $c(v_m, v_n)$ was proposed to study the relationship between v_m and v_n [83]. It is defined as

$$c\left(v_m, v_n\right) = \left\langle \frac{\left(v_m - \left\langle v_m \right\rangle_{ev}\right)\left(v_n - \left\langle v_n \right\rangle_{ev}\right)}{\sigma_{v_m}\sigma_{v_n}} \right\rangle_{ev}, \quad (3)$$

where σ_{v_m} is the standard deviation of the quantity v_m; $c(v_m, v_n)$ is 1 (or −1) if v_m and v_n are linearly (antilinearly) correlated and is 0 if they are not correlated. It was shown in Figure 1 that there is an anticorrelation between v_2 and

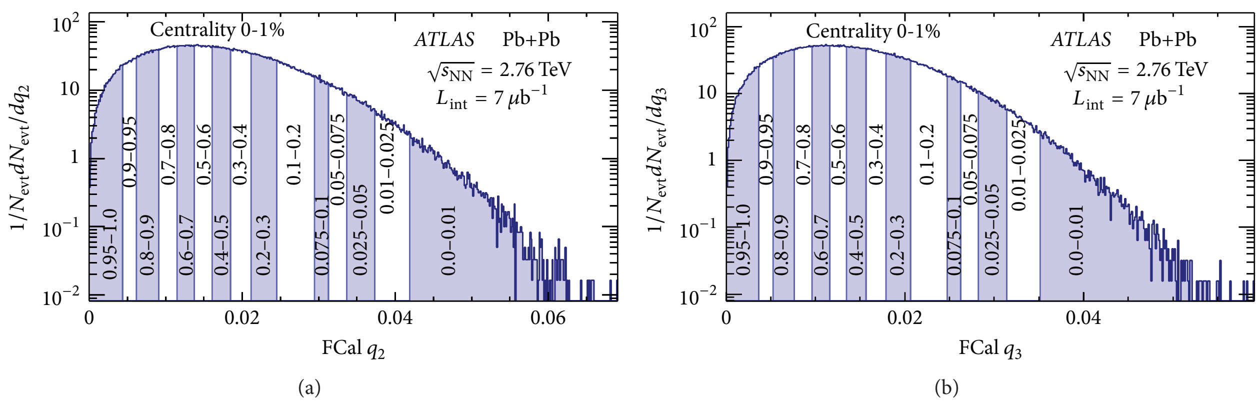

Figure 2: Distributions of V_2 (a) and V_3 (b) calculated with ATLAS forward calorimeter for centrality interval 0-1%. Figures are taken from [45].

v_3, while a correlation was observed between v_2 and v_4. In addition, it was demonstrated that $c(v_2, v_4)$ depends on both the initial conditions and η/s, while $c(v_2, v_3)$ is only sensitive to η/s [83]. Nevertheless, it cannot be accessible easily in experimental measurements, which rely on two-particle and multiparticle correlations techniques. Thus, it is critical to find an observable which studies the relationship between flow harmonics without contributions from symmetry plane correlations and can be accessed with observable techniques from experiments. Two different approaches, named *Event Shape Engineering* and *Symmetric Cumulant*, are discussed in the following section.

2.1. Event Shape Engineering (ESE). The first experimental attempt was made by ATLAS Collaboration [45], using the Event Shape Engineering (ESE) [96]. This is a technique to select events according to the magnitude of reduced flow vector $\overrightarrow{V_n}$. Figure 2 shows the performance of event shape selection on V_2 (a) and V_3 (b) in ATLAS detector. For each centrality the data sample is divided into several event classes according to V_2 or V_3 distributions. Then v_2 and v_3 relationship was investigated by measurements of v_2 and v_3 in each event class from ESE selection. Without using ESE selection, a boomerang-like pattern was observed for the centrality dependence of v_2-v_3 correlation. This is mainly due to the fact that v_3 has weaker centrality dependence than v_2. By using ESE, it was observed in Figure 3(b) that, for event class with the same centrality (shown as the same color), v_3 decreases as v_2 increases. It suggests that v_2 is anticorrelated with v_3. Considering the linear hydrodynamic response of v_2 and v_3 from eccentricity ε_2 and triangularity ε_3, the anticorrelation between v_2 and v_3 might reveal the anticorrelation between ε_2 and ε_3 of the initial geometry. This indication of initial anticorrelations between ε_2 and ε_3 is observed in model calculations [96, 97].

Figure 4 shows the investigation of relationship between v_2 and v_4. A boomerang-like pattern, although weaker than that for v_2-v_3 relationship shown in Figure 3(a), is observed in Figure 4(a), prior to the ESE selection. After the ESE selection, it is found in Figure 4(b) that v_4 increases with increasing v_2. This suggests a correlation between the two harmonics and it can be understood by the interplay between linear and nonlinear collective dynamics in the system evolution [45]. This nonlinear contribution of v_4 from v_2 is further investigated by fitting the correlation pattern using $v_4 = \sqrt{c_0^2 + (c_1 v_2^2)^2}$, where c_0 and c_1 denote the linear and nonlinear components. It is found that the linear component has weak centrality dependence, while the nonlinear component, increasing dramatically with collision centrality, becomes the dominant contribution in the most peripheral collisions [45].

These (anti)correlation patterns between v_m and v_n observed in experiments open a new window to the understanding of the collectivity phenomena in heavy-ion collisions. However, it was also noticed that these measurements were based on 2-particle correlations, which might be suffered by nonflow effects, and they require subdividing such calculations and modeling resolutions associated with ESE due to finite event-wise multiplicities. Considering the computational constraints, this approach cannot be performed easily in hydrodynamic calculations which usually are based on limited statistics compared to experimental data.

2.2. Symmetric Cumulants (SC). A new type of observables for the analyses of flow harmonic correlations, *symmetric cumulants* (originally named *Standard Candles* (SC) in [93]), was proposed as $\mathrm{SC}(m,n) = \langle\langle \cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4)\rangle\rangle_c$. If $m \neq n$, the isotropic part of the corresponding four-particle cumulant is given by

$$\begin{aligned}
&\langle\langle \cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4)\rangle\rangle_c \\
&\quad = \langle\langle \cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4)\rangle\rangle \\
&\quad\quad - \langle\langle \cos[m(\varphi_1 - \varphi_2)]\rangle\rangle \langle\langle \cos[n(\varphi_1 - \varphi_2)]\rangle\rangle \\
&\quad = \langle v_m^2 v_n^2\rangle - \langle v_m^2\rangle \langle v_n^2\rangle .
\end{aligned} \tag{4}$$

For a detector with uniform acceptance in azimuthal direction, the asymmetric terms, for example, $\langle\langle \cos(m\varphi_1 - n\varphi_2)\rangle\rangle$,

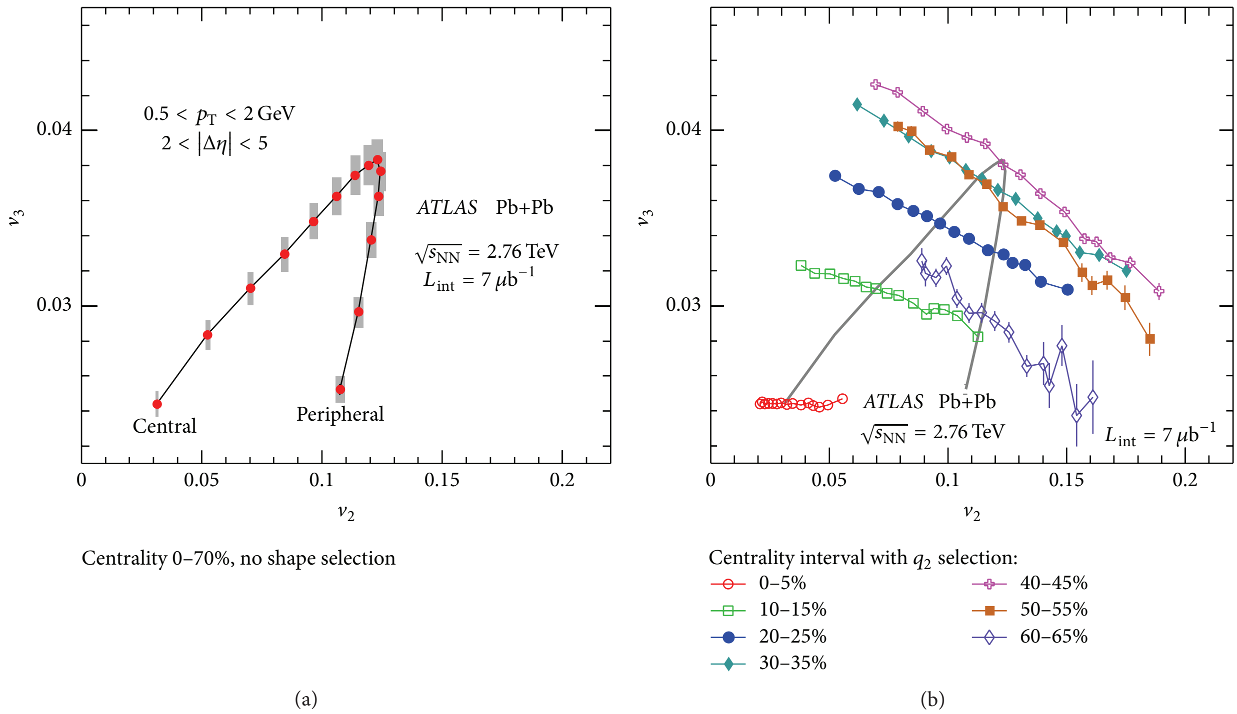

FIGURE 3: The correlation of v_2 (x-axis) with v_3 (y-axis) measured in $0.5 < p_T < 2$ GeV/c. (a) shows v_2 and v_3 values for fourteen 5% centrality intervals over the centrality range of 0–70% without event shape selection. (b) shows v_2 and v_3 values in 15 q_2 intervals in seven centrality ranges (markers) with larger v_2 value corresponding to larger q_2 value. Figures are taken from [45].

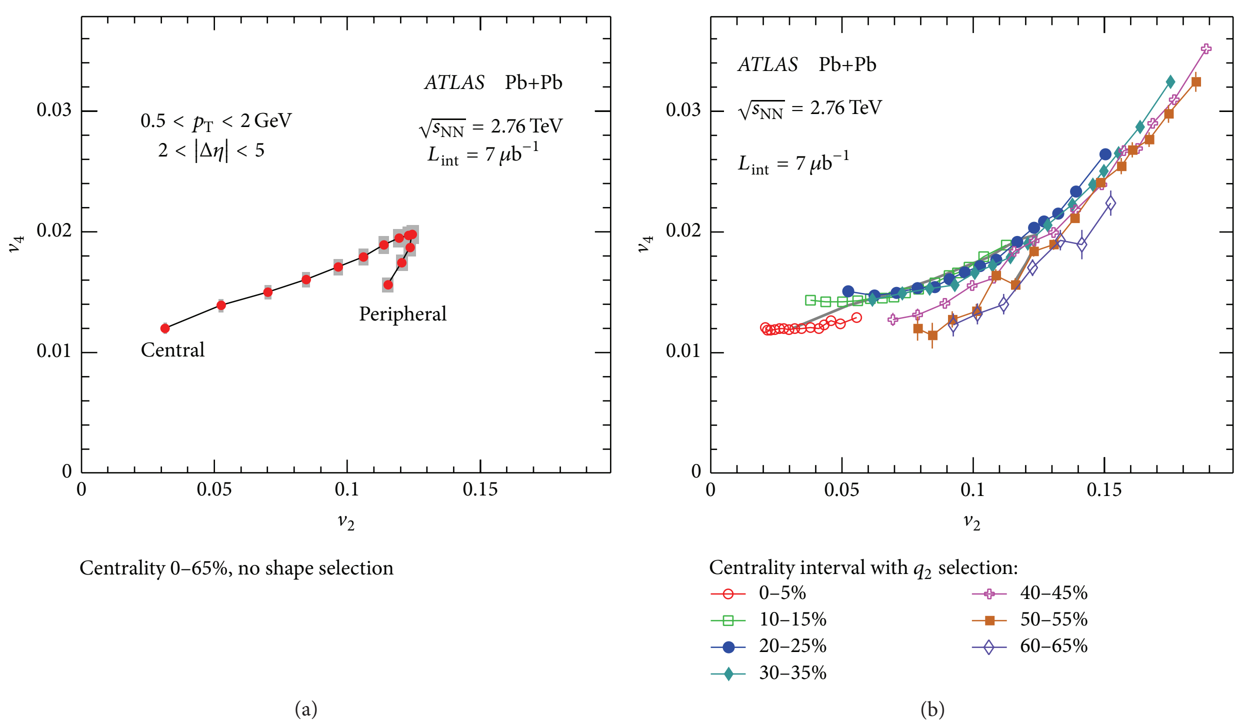

FIGURE 4: The correlation of v_2 (x-axis) with v_4 (y-axis) measured in $0.5 < p_T < 2$ GeV/c. (a) shows v_2 and v_3 values for fourteen 5% centrality intervals over the centrality range of 0–70% without event shape selection. (b) shows v_2 and v_4 values in 15 q_2 intervals in seven centrality ranges (markers) with larger v_2 value corresponding to larger q_2 value. Figures are taken from [45].

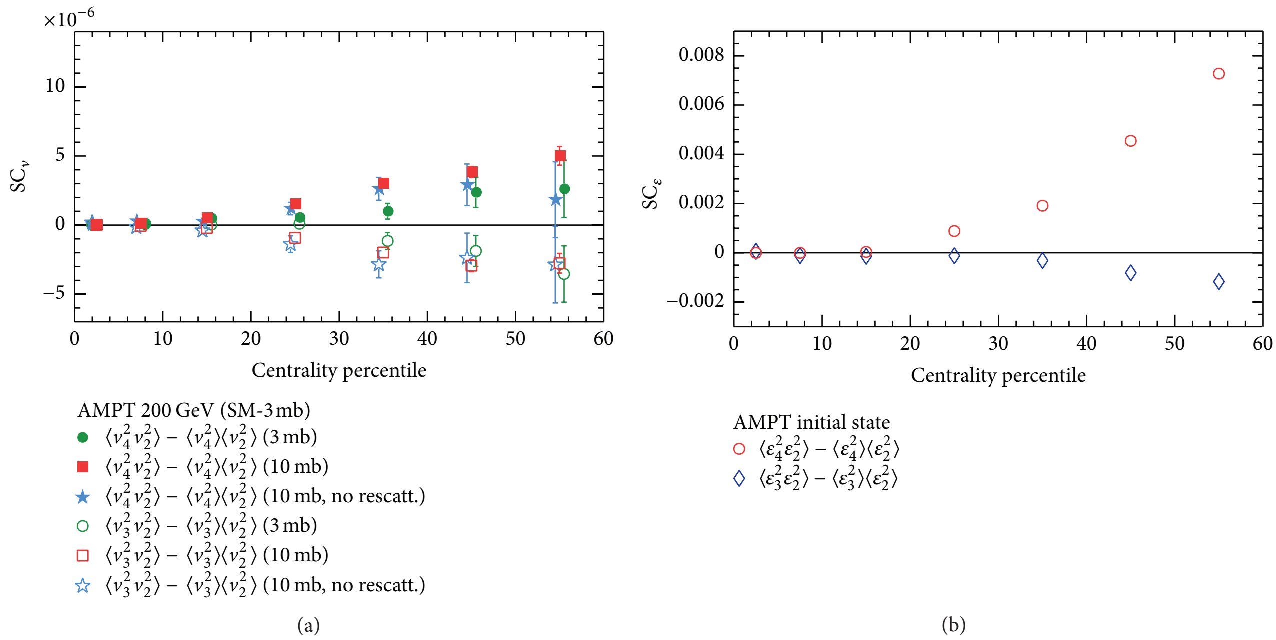

FIGURE 5: The centrality dependence of symmetric cumulants SC(4, 2) and SC(3, 2) at $\sqrt{s_{\mathrm{NN}}}$. Figures are taken from [72, 93].

are averaged to zero. The single event 4-particle correlation $\langle\langle\cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4)\rangle\rangle$ could be calculated as

$$\begin{aligned}
&\langle \cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4)\rangle \\
&= \frac{1}{M(M-1)(M-2)(M-3)}\left[|V_m|^2|V_n|^2\right. \\
&- 2\mathfrak{Re}\left[V_{m+n}V_m^*V_n^*\right] - 2\mathfrak{Re}\left[V_mV_{m-n}^*V_n^*\right] + |V_{m+n}|^2 \\
&\left. + |V_{m-n}|^2 - (M-4)\left(|V_m|^2 + |V_n|^2\right) + M(M-6)\right].
\end{aligned} \tag{5}$$

And the single event 2-particle correlation $\langle\langle\cos[m(\varphi_1 - \varphi_2)]\rangle\rangle$ could be obtained as

$$\langle \cos[m(\varphi_1 - \varphi_2)]\rangle = \frac{1}{M(M-1)}\left[|V_m|^2 - M\right]. \tag{6}$$

Then, the weights of $M(M-1)$ and $M(M-1)(M-2)(M-3)$ are used to get the event-averaged 2-particle and 4-particle correlations, as introduced in [93]. Due to the definition, this new type of 4-particle cumulants SC(m,n) is independent of the symmetry planes Ψ_m and Ψ_n and is expected to be less sensitive to nonflow correlations, which should be strongly suppressed in 4-particle cumulants. This was confirmed by SC(m,n) calculation using HIJING model [98, 99] which does not include anisotropic collectivity but, for example, azimuthal correlations due to jet production. It is observed that both $\langle\langle\cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4)\rangle\rangle$ and $\langle\langle\cos[m(\varphi_1 - \varphi_2)]\rangle\rangle\langle\langle\cos[n(\varphi_1 - \varphi_2)]\rangle\rangle$ are nonzero, while SC(m,n) are compatible with zero in HIJING simulations [36]. This confirms that SC(m,n) measurements are nearly insensitive to nonflow correlations. Therefore, it is believed that SC(m,n) is nonzero if there is (anti)correlations of v_n and v_m. The investigation of SC(m,n) will allow us to know whether finding v_m larger than $\langle v_m\rangle$ in an event will enhance or reduce the probability of finding v_n larger than $\langle v_n\rangle$ in that event, which provides unique information for the event-by-event simulations of anisotropic flow harmonics.

Figure 5 shows the first calculation of SC(4, 2) (solid markers) and SC(3, 2) (open markers) as a function of centrality from AMPT model [93]. Nonzero values for both SC(4, 2) and SC(3, 2) are observed. Positive SC(4, 2) suggests a correlation between the event-by-event fluctuations of v_2 and v_4, which indicates that finding v_2 larger than $\langle v_2\rangle$ in an event enhances the probability of finding v_4 larger than $\langle v_4\rangle$ in that event. On the other hand, the negative results of SC(3, 2) imply that finding v_2 larger than $\langle v_2\rangle$ enhances the probability of finding v_3 smaller than $\langle v_3\rangle$ [93].

Several configurations of the AMPT model have been investigated to better understand the results based on AMPT simulations [93]. Partonic interactions can be tweaked by changing the partonic cross section: the default value is 10 mb, while using 3 mb generates weaker partonic interactions in ZPC [100, 101]. One can also change the hadronic interactions by controlling the termination time in ART. Setting NTMAX = 3, where NTMAX is a parameter which controls the number of time steps in ART (rescattering time), will effectively turn off the hadronic interactions [100, 101]. For SC(4, 2) and SC(3, 2) calculations for three different scenarios, (a) 3 mb, (b) 10 mb, and (c) 10 mb, no rescattering is presented in Figure 5. It is found that when the partonic cross section is decreasing from 10 mb (lower shear viscosity) to 3 mb (higher shear viscosity), the strength of SC(4, 2) decreases. Additionally, the "10 mb, no rescattering" setup seems to give slightly smaller magnitudes of SC(4, 2) and SC(3, 2).

Further studies have been performed in AMPT initial conditions, based on the observable of SC$(m,n)_\varepsilon$ which is defined as $\langle\varepsilon_m^2\varepsilon_n^2\rangle - \langle\varepsilon_m^2\rangle\langle\varepsilon_n^2\rangle$ [72]. The centrality dependence

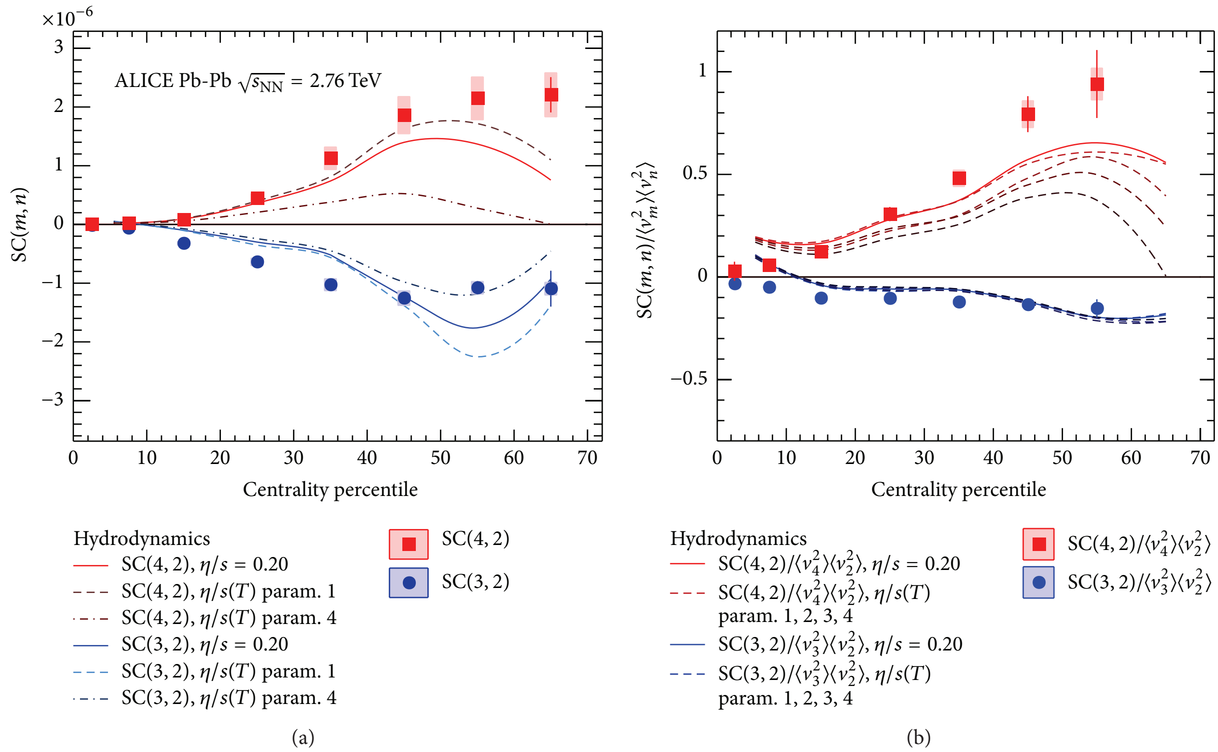

Figure 6: The centrality dependence of symmetric cumulants SC(4, 2) (red markers) and SC(3, 2) (blue markers) at $\sqrt{s_{NN}} = 2.76$ TeV Pb–Pb collisions. Figures are taken from [36].

of $SC(4,2)_\varepsilon$ and $SC(3,2)_\varepsilon$ is presented as red circles and blue diamonds in Figure 5(b). Positive and increasing trend from central to peripheral collisions has been observed for $SC(4,2)_\varepsilon$. In contrast, negative and decreasing trend was observed for $SC(3,2)_\varepsilon$ in the AMPT initial conditions. This shows that finding ε_2 larger than $\langle\varepsilon_2\rangle$ in an event enhances the probability of finding ε_4 larger than $\langle\varepsilon_4\rangle$, while in parallel enhancing the probability of finding ε_3 smaller than $\langle\varepsilon_3\rangle$ in that event. Same conclusions were obtained using MC-Glauber initial conditions [75].

Based on AMPT calculations, it seems that the signs of $SC(m,n)_v$ (for $m,n = 2,3,4$) in the final state are determined by the correlations of $SC(m,n)_\varepsilon$ in the initial state, while its magnitude also depends on the properties of the created system. This clearly suggests that $SC(m,n)_v$ is a new promising observable to constrain the initial conditions and the transport properties of the system.

The first experimental measurements of centrality dependence of SC(4, 2) (red squares) and SC(3, 2) (blue circles) are presented in Figure 6(a). Positive values of SC(4, 2) are observed for all cases of centrality. This confirms a correlation between the event-by-event fluctuations of v_2 and v_4. On the other hand, the measured negative results of SC(3, 2) show the anticorrelation between v_2 and v_3 magnitudes. The same measurements are performed using the like-sign technique, which is another powerful approach to estimate nonflow effects [27]. It was found that the difference between correlations for like-sign and all charged combinations, which might be mainly due to nonflow effects, is much smaller compared to the magnitudes of $SC(m,n)$ itself. This further proves that nonzero values of $SC(m,n)$ measured in experiments cannot be explained by nonflow effects solely.

In addition, the comparison between experimental data and the event-by-event perturbative-QCD+saturation+hydro ("EKRT") calculations [62], which incorporate both initial conditions and hydrodynamic evolution, is shown in Figure 6. It was shown that this model can capture quantitatively the centrality dependence of individual v_2, v_3, and v_4 harmonics in central and mid-central collisions [62]. However, it can only qualitatively but not quantitatively predict $SC(m,n)$ measurements by ALICE. For given $\eta/s(T)$ parameterization tuned by individual flow harmonic, the calculation cannot describe SC(4, 2) and SC(3, 2) simultaneously for any single centrality. Experimental measurements are also compared to the VISH2+1 model calculations (see Figure 7), using various combinations of initial conditions (IC) from (a) MC-Glb, (b) MC-KLN, and (c) MC-AMPT with $\eta/s = 0.08$ and 0.20. It is noticed that the one with MC-Glb IC and $\eta/s = 0.08$ is compatible with SC(4, 2) measurement and the calculation with MC-AMPT IC and $\eta/s = 0.08$ can describe SC(3, 2) measurement [70]. However, just like EKRT calculations, none of these combinations is able to describe SC(4, 2) and SC(3, 2) simultaneously. Thus, it is concluded that the new $SC(m,n)$ observables provide better handle on the initial conditions and $\eta/s(T)$ than each of the individual harmonic measurements alone.

After being presented for the first time at Quark Matter 2015 conference, preliminary results of SC(4, 2) and SC(3, 2)

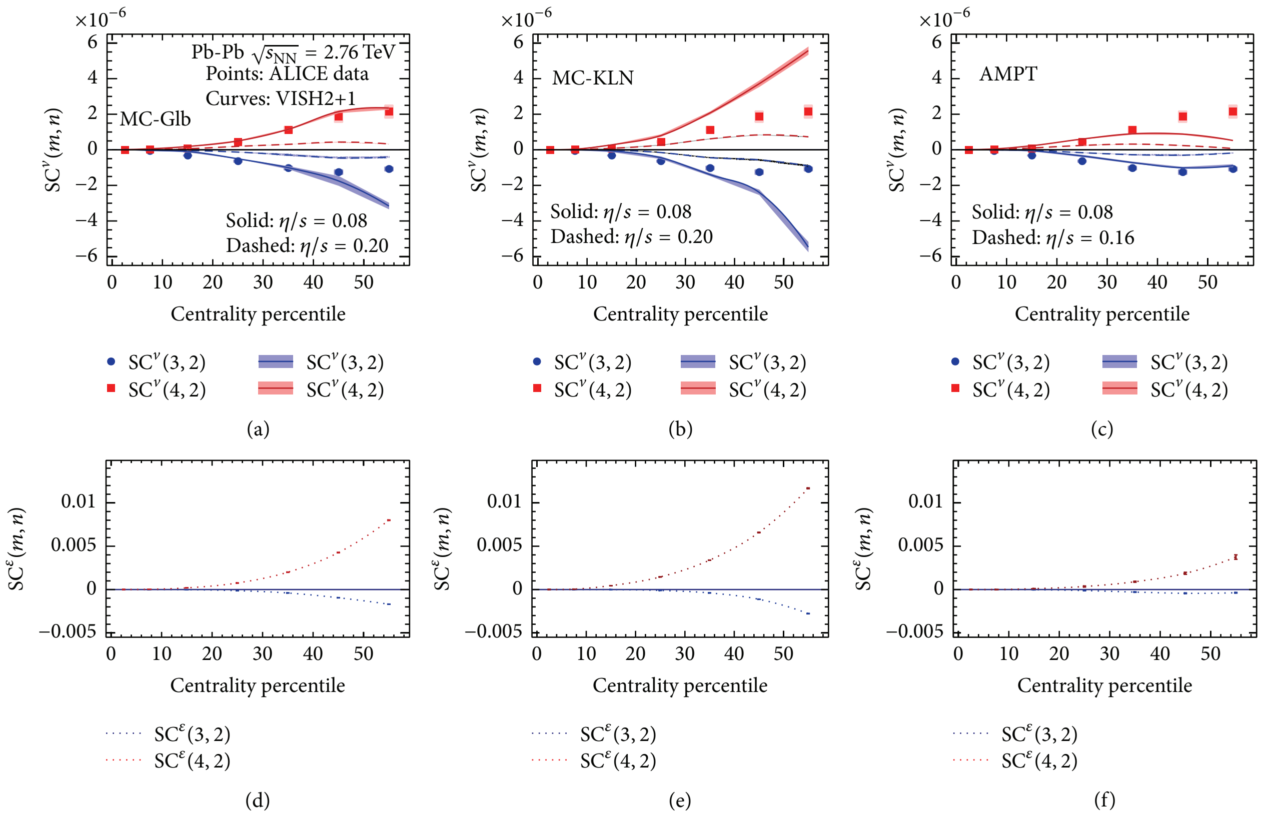

Figure 7: The centrality dependence of symmetric cumulants SC(4, 2) (red markers) and SC(3, 2) (blue markers) at $\sqrt{s_{NN}}$ = 2.76 TeV Pb–Pb collisions by VISH2+1 simulations. Figures are taken from [70].

gained a lot of attention [102]. One of the key suggestions was to normalize $SC(m,n)$ by dividing with the products $\langle v_m^2\rangle\langle v_n^2\rangle$ in order to get rid of influences from individual flow harmonics. The results are shown in Figure 6(b), with normalized SC(3, 2) and SC(4, 2) observables by dividing with the products $\langle v_3^2\rangle\langle v_2^2\rangle$ and $\langle v_4^2\rangle\langle v_2^2\rangle$, respectively [36]. The 2-particle correlations $\langle v_m^2\rangle$ and $\langle v_n^2\rangle$ are obtained with a pseudorapidity gap of $|\Delta\eta| > 1.0$ to suppress contributions from nonflow effects. It was shown in Figure 8(a) that the normalized SC(4, 2) observable exhibits clear sensitivity to different η/s parameterizations and the initial conditions, which provides a unique opportunity to discriminate between various possibilities of the detailed setting of $\eta/s(T)$ of the produced QGP and the initial conditions used in hydrodynamic calculations. On the other hand, normalized SC(3, 2) is independent of the setting of $\eta/s(T)$. In addition, it was demonstrated in Figure 9 that the normalized SC(3, 2), also named $NSC^v(3,2)$ in the following text, is compatible with its corresponding observable $SC^{\varepsilon}(3,2)$ in the initial state. Thus, $NSC^v(3,2)$ could be taken as golden observable to directly constrain initial conditions without demands for precise knowledge of transport properties of the system [70]. Furthermore, none of existing theoretical calculations can reproduce the data; there is still a long way to go for the development of hydrodynamic calculations.

Predictions of relationship between other harmonics are provided in [70] and shown in Figure 8. Besides different sensitivities to IC and η/s as seen above, the centrality dependence of the relationship between flow harmonics seems quite different. For instance, despite the differences in the initial conditions, a maximum value of SC(5, 3) is observed in central collision using η/s = 0.20, while the maximum value is seen in more peripheral collision if η/s = 0.08 is used.

Compared to the previous measurements of relationship between flow harmonics investigated using the ESE technique, $SC(m,n)$ observable provides a quantitative measure of these correlation strengths. Further investigations on relationship between flow harmonics using list of observables in Table 1 could be performed as a function of centrality, transverse momentum, and pseudorapidity et al., which is clearly nontrivial. Although one did not use the information of symmetry planes in both ESE and SC studies, recent study just reveals that flow harmonic correlations might not be completely independent on symmetry plane correlations [73]. The proportionality relations between symmetric cumulants involving higher harmonics v_4 or v_5 and symmetry plane correlations are derived, which seem to build the bridge between flow harmonic correlations and flow angle correlations (symmetry plane correlations). This might point out to a new direction of investigations of correlations between flow vectors and will shed a new light on the nature of fluctuating initial conditions and η/s of the created QGP in heavy-ion collisions.

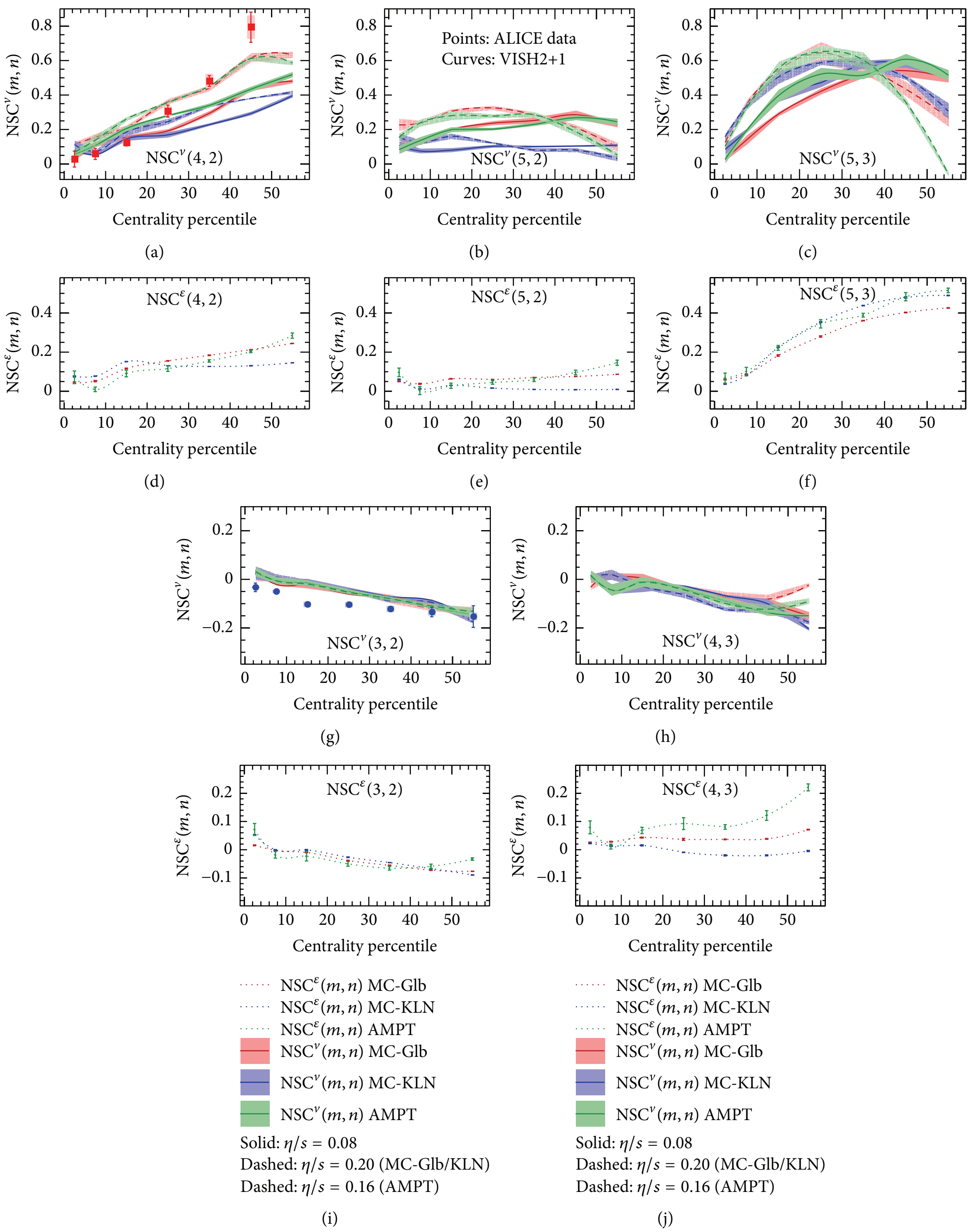

FIGURE 8: The centrality dependence of normalized symmetric cumulants NSC(m,n) at $\sqrt{s_{NN}} = 2.76$ TeV Pb–Pb collisions by VISH2+1 simulations. Figures are taken from [70].

3. Summary

In the past two decades, the underlying PDF of each single harmonic $P(v_n)$ was investigated in great detail. However, at the moment, how the joint underlying PDF, including different order symmetry planes and harmonics, is described is an open question, especially if these correlations between different flow harmonics modify the single harmonics $P(v_n)$. New observables discussed here begin to answer these open questions. Nevertheless, many more investigations between

Table 1: List of observables for correlations of flow harmonics, including all combinations of symmetric 2-harmonic 4-particle cumulants (up to v_6).

Observables	Equations	Number of particles	Exp.	Th.
$\langle\langle\cos(2\varphi_1+3\varphi_2-2\varphi_3-3\varphi_4)\rangle\rangle_c$	$\langle v_2^2v_3^2\rangle-\langle v_2^2\rangle\langle v_3^2\rangle$	4	[36]	[70–72]
$\langle\langle\cos(2\varphi_1+4\varphi_2-2\varphi_3-4\varphi_4)\rangle\rangle_c$	$\langle v_2^2v_4^2\rangle-\langle v_2^2\rangle\langle v_4^2\rangle$	4	[36]	[70–73]
$\langle\langle\cos(2\varphi_1+5\varphi_2-2\varphi_3-5\varphi_4)\rangle\rangle_c$	$\langle v_2^2v_5^2\rangle-\langle v_2^2\rangle\langle v_5^2\rangle$	4		[70, 71, 73]
$\langle\langle\cos(2\varphi_1+6\varphi_2-2\varphi_3-6\varphi_4)\rangle\rangle_c$	$\langle v_2^2v_6^2\rangle-\langle v_2^2\rangle\langle v_6^2\rangle$	4		
$\langle\langle\cos(3\varphi_1+4\varphi_2-3\varphi_3-4\varphi_4)\rangle\rangle_c$	$\langle v_3^2v_4^2\rangle-\langle v_3^2\rangle\langle v_4^2\rangle$	4		[70]
$\langle\langle\cos(3\varphi_1+5\varphi_2-3\varphi_3-5\varphi_4)\rangle\rangle_c$	$\langle v_3^2v_5^2\rangle-\langle v_3^2\rangle\langle v_5^2\rangle$	4		[70, 71, 73]
$\langle\langle\cos(3\varphi_1+6\varphi_2-3\varphi_3-6\varphi_4)\rangle\rangle_c$	$\langle v_3^2v_6^2\rangle-\langle v_3^2\rangle\langle v_6^2\rangle$	4		
$\langle\langle\cos(4\varphi_1+5\varphi_2-4\varphi_3-5\varphi_4)\rangle\rangle_c$	$\langle v_4^2v_5^2\rangle-\langle v_4^2\rangle\langle v_5^2\rangle$	4		
$\langle\langle\cos(4\varphi_1+6\varphi_2-4\varphi_3-6\varphi_4)\rangle\rangle_c$	$\langle v_4^2v_6^2\rangle-\langle v_4^2\rangle\langle v_6^2\rangle$	4		
$\langle\langle\cos(5\varphi_1+6\varphi_2-5\varphi_3-6\varphi_4)\rangle\rangle_c$	$\langle v_5^2v_6^2\rangle-\langle v_5^2\rangle\langle v_6^2\rangle$	4		
...	...	6		

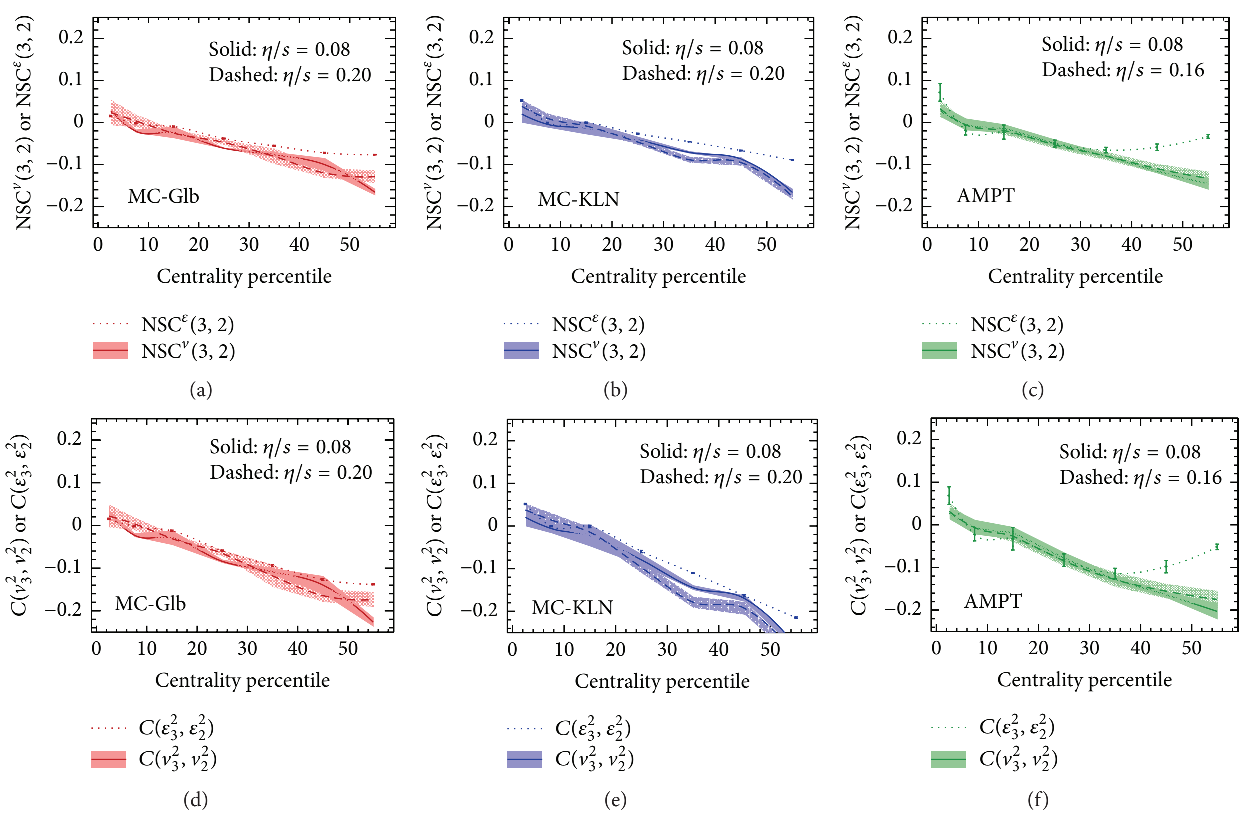

Figure 9: The centrality dependence of NSC(3, 2) (a, b, and c) and $C(v_3^2, v_2^2)$ (d, e, and f) and the corresponding observables in the initial conditions at $\sqrt{s_{NN}}$ = 2.76 TeV Pb–Pb collisions from VISH2+1. Figures are taken from [70].

different flow harmonics, including higher-order cumulants and higher harmonics, are necessary to reasonably constrain the joint PDF and ultimately lead to new insights into the nature of fluctuation of the created matter in heavy-ion collisions. How to turn the multitude of measured and possibly measurable in future relationships between anisotropic flow harmonics into a focused search for correct initial conditions and detailed setting of η/s is an exciting challenge for the theory community.

Competing Interests

The author declares that there are no competing interests regarding the publication of this paper.

Acknowledgments

The author thanks J. J. Gaardhøje, K. Gajdošová, L. Yan, J. Y. Ollitrault, and H. Song for the comments on the manuscript and fruitful discussions. The author is supported by the Danish Council for Independent Research, Natural Sciences, and the Danish National Research Foundation (Danmarks Grundforskningsfond).

References

[1] T. D. Lee, "Feynman rules of quantum chromodynamics inside a hadron," *Physical Review D*, vol. 19, no. 6, pp. 1802–1819, 1979.

[2] E. V. Shuryak, "Quantum chromodynamics and the theory of superdense matter," *Physics Reports*, vol. 61, no. 2, pp. 71–158, 1980.

[3] J.-Y. Ollitrault, "Anisotropy as a signature of transverse collective flow," *Physical Review D*, vol. 46, no. 1, pp. 229–245, 1992.

[4] S. Voloshin and Y. Zhang, "Flow study in relativistic nuclear collisions by Fourier expansion of azimuthal particle distributions," *Zeitschrift fur Physik C-Particles and Fields*, vol. 70, no. 4, pp. 665–671, 1996.

[5] C. Alt, T. Anticic, B. Baatar et al., "Directed and elliptic flow of charged pions and protons in Pb + Pb collisions at 40A and 158 *A* GeV," *Physical Review C*, vol. 68, no. 3, Article ID 034903, 2003.

[6] K. H. Ackermann, N. Adams, C. Adler et al., "Elliptic flow in *Au + Au* collisions at $\sqrt{s_{\mathrm{NN}}}$ = 130 GeV," *Physical Review Letters*, vol. 86, pp. 402–407, 2001.

[7] C. Adler, Z. Ahammed, C. Allgower et al., "Identified particle elliptic flow in Au + Au collisions at $\sqrt{s_{NN}}$ = 130 GeV," *Physical Review Letters*, vol. 87, no. 18, Article ID 182301, 2001.

[8] C. Adler, Z. Ahammed, Z. Allgower et al., "Elliptic flow from two- and four-particle correlations in Au+Au collisions at $\sqrt{s_{\mathrm{NN}}}$ = 130 GeV," *Physical Review C*, vol. 66, Article ID 034904, 2002.

[9] J. Adams, M. M. Aggarwal, Z. Ahammed et al., "Azimuthal anisotropy in Au+Au collisions at $\sqrt{s_{\mathrm{NN}}}$ = 200 GeV," *Physical Review C*, vol. 72, no. 1, Article ID 014904, 23 pages, 2005.

[10] J. Adams, L. Adamczyk, J. K. Adkins et al., "Multistrange Baryon Elliptic Flow in Au + Au Collisions at $\sqrt{s_{NN}}$ = 200 GeV," *Physical Review Letters*, vol. 95, no. 12, Article ID 122301, 2005.

[11] B. I. Abelev, M. M. Aggarwal, Z. Ahammed et al., "Centrality dependence of charged hadron and strange hadron elliptic flow from $\sqrt{s_{NN}}$ = 200 GeV Au+Au collisions," *Physical Review C*, vol. 77, Article ID 054901, 2008.

[12] L. Adamczyk, J. K. Adkins, G. Agakishiev et al., "Elliptic flow of identified hadrons in Au+Au collisions at $\sqrt{s_{\mathrm{NN}}}$ = 7.7–62.4 GeV," *Physical Review C*, vol. 88, no. 1, Article ID 014902, 25 pages, 2013.

[13] K. Adcox, S. S. Adler, N. N. Ajitanand et al., "Flow measurements via two-particle azimuthal correlations in Au+Au collisions at $\sqrt{s_{\mathrm{NN}}}$ = 130 GeV," *Physical Review Letters*, vol. 89, no. 21, Article ID 212301, 2002.

[14] S. S. Adler, S. Afanasiev, C. Aidala et al., "Elliptic flow of identified hadrons in Au+Au collisions at $\sqrt{s_{NN}}$ = 200 GeV," *Physical Review Letters*, vol. 91, no. 18, Article ID 182301, 2003.

[15] A. Adare, S. Afanasiev, C. Aidala et al., "Scaling properties of azimuthal anisotropy in Au + Au and Cu + Cu collisions at $\sqrt{s_{\mathrm{NN}}}$ = 200 GeV," *Physical Review Letters*, vol. 98, Article ID 162301, 2007.

[16] S. Afanasiev, C. Aidala, N. N. Ajitanand et al., "Systematic studies of elliptic flow measurements in Au + Au collisions at $\sqrt{s_{\mathrm{NN}}}$ = 200 GeV," *Physical Review C*, vol. 80, no. 2, Article ID 024909, 2009.

[17] A. Adare, S. Afanasiev, C. Aidala et al., "Elliptic and hexadecapole flow of charged hadrons in Au+Au collisions at $\sqrt{s_{\mathrm{NN}}}$ = 200 GeV," *Physical Review Letters*, vol. 105, no. 6, Article ID 062301, 6 pages, 2010.

[18] A. Adare, S. Afanasiev, C. Aidala et al., "Measurements of higher order flow harmonics in Au + Au collisions at $\sqrt{s_{NN}}$ = 200 GeV," *Physical Review Letters*, vol. 107, Article ID 252301, 2011.

[19] B. B. Back, M. D. Baker, D. S. Barton et al., "Pseudorapidity and centrality dependence of the collective flow of charged particles in Au + Au collisions at $\sqrt{s_{NN}}$ = 200 GeV," *Physical Review Letters*, vol. 89, no. 22, Article ID 222301, 2002.

[20] B. B. Back, M. D. Baker, M. Ballintijn et al., "Centrality and pseudorapidity dependence of elliptic flow for charged hadrons in Au+Au collisions at $\sqrt{s_{NN}}$ = 200 GeV," *Physical Review C*, vol. 72, Article ID 051901, 2005.

[21] B. B. Back, M. D. Baker, M. Ballintijn et al., "Energy dependence of elliptic flow over a large pseudorapidity range in Au+Au collisions at the BNL relativistic heavy ion collider," *Physical Review Letters*, vol. 94, no. 12, Article ID 122303, 4 pages, 2005.

[22] S. Manly, B. Alver, B. B. Back et al., "System size, energy and pseudorapidity dependence of directed and elliptic flow at RHIC," *Nuclear Physics A*, vol. 774, pp. 523–526, 2006.

[23] B. B. Back, M. D. Baker, M. Ballintijn et al., "Energy dependence of directed flow over a wide range of pseudorapidity in Au + Au collisions at the BNL relativistic heavy ion collider," *Physical Review Letters*, vol. 97, no. 1, Article ID 012301, 2006.

[24] B. Alver, B. B. Back, M. D. Baker et al., "System size, energy, pseudorapidity, and centrality dependence of elliptic flow," *Physical Review Letters*, vol. 98, no. 24, Article ID 242302, 5 pages, 2007.

[25] B. Alver, B. B. Back, M. D. Baker et al., "Event-by-event fluctuations of azimuthal particle anisotropy in Au + Au collisions at $\sqrt{s_{NN}}$ = 200 GeV," *Physical Review Letters*, vol. 104, no. 14, Article ID 142301, 2010.

[26] B. Alver, B. B. Back, M. D. Baker et al., "Non-flow correlations and elliptic flow fluctuations in Au+Au collisions at $\sqrt{s_{\mathrm{NN}}}$ = 200 GeV," *Physical Review C*, vol. 81, no. 3, Article ID 034915, 8 pages, 2010.

[27] K. Aamodt, B. Abelev, A. Abrahantes Quintana et al., "Elliptic flow of charged particles in Pb-Pb collisions at $\sqrt{s_{NN}}$ = 2.76 TeV," *Physical Review Letters*, vol. 105, no. 25, Article ID 252302, 2010.

[28] K. Aamodt, B. Abelev, A. Abrahantes et al., "Higher harmonic anisotropic flow measurements of charged particles in Pb-Pb collisions at $\sqrt{s_{NN}}$ = 2.76 TeV," *Physical Review Letters*, vol. 107, no. 3, Article ID 032301, 2011.

[29] B. Abelev, J. Adam, D. Adamová et al., "Anisotropic flow of charged hadrons, pions and (anti-)protons measured at high transverse momentum in Pb-Pb collisions at $\sqrt{s_{\mathrm{NN}}}$ = 2.76 Tev," *Physics Letters B*, vol. 719, no. 1–3, pp. 18–28, 2013.

[30] B. Abelev, J. Adam, D. Adamová et al., "Directed flow of charged particles at midrapidity relative to the spectator plane in Pb-Pb collisions at $\sqrt{s_{NN}}$ = 2.76 TeV," *Physical Review Letters*, vol. 111, no. 23, Article ID 232302, 2013.

[31] B. B. Abelev, J. Adam, D. Adamová et al., "Multiparticle azimuthal correlations in *p*-Pb and Pb-Pb collisions at the CERN large hadron collider," *Physical Review C*, vol. 90, no. 5, Article ID 054901, 2014.

[32] B. B. Abelev, J. Adam, D. Adamová et al., "Elliptic flow of identified hadrons in Pb-Pb collisions at $\sqrt{s_{\mathrm{NN}}}$ = 2.76 Tev," *Journal of High Energy Physics*, vol. 2015, no. 6, article 190, pp. 1–41, 2015.

[33] J. Adam, D. Adamová, M. M. Aggarwal et al., "Event-shape engineering for inclusive spectra and elliptic flow in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV," *Physical Review C*, vol. 93, no. 3, Article ID 034916, 2016.

[34] J. Adam, L. Aphecetche, B. Audurier et al., "Charge-dependent flow and the search for the chiral magnetic wave in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV," *Physical Review C*, vol. 93, no. 4, Article ID 044903, 2016.

[35] J. Adam, D. Adamová, M. M. Aggarwal et al., "Anisotropic flow of charged particles in Pb-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV," *Physical Review Letters*, vol. 116, no. 13, Article ID 132302, 12 pages, 2016.

[36] J. Adam, J. Milosevic, G. Biro et al., "Correlated event-by-event fluctuations of flow harmonics in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV," https://arxiv.org/abs/1604.07663.

[37] G. Aad, B. Abbott, J. Abdallah et al., "Measurement of the pseudorapidity and transverse momentum dependence of the elliptic flow of charged particles in lead–lead collisions at $\sqrt{S_{NN}} = 2.76$ TeV with the ATLAS detector," *Physics Letters B*, vol. 707, no. 3-4, pp. 330–348, 2012.

[38] G. Aad, B. Abbott, J. Abdallah et al., "Measurement of the azimuthal anisotropy for charged particle production in $\sqrt{s_{\mathrm{NN}}} = 2.76$ Tev lead-lead collisions with the ATLAS detector," *Physical Review C*, vol. 86, no. 1, Article ID 014907, 41 pages, 2012.

[39] G. Aad, T. Abajyan, B. Abbott et al., "Measurement with the ATLAS detector of multi-particle azimuthal correlations in p+Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV," *Physics Letters B*, vol. 725, no. 1–3, pp. 60–78, 2013.

[40] G. Aad, T. Tatevik, B. Abbott et al., "Measurement of the distributions of event-by-event flow harmonics in lead-lead collisions at = 2.76 TeV with the ATLAS detector at the LHC," *JHEP*, vol. 1311, p. 183, 2013.

[41] G. Aad, E. Abat, B. Abbott et al., "Measurement of event-plane correlations in $\sqrt{S_{NN}} = 2.76$ TeV lead-lead collisions with the ATLAS detector," *Physical Review C*, vol. 90, no. 2, Article ID 024905, 2014.

[42] G. Aad, B. Abbott, J. Abdallah et al., "Measurement of the centrality and pseudorapidity dependence of the integrated elliptic flow in lead–lead collisions at $\sqrt{s_{NN}} = 2.76$ TeV with the ATLAS detector," *The European Physical Journal C*, vol. 74, article 2982, 2014.

[43] G. Aad, B. Abbott, J. Abdallah et al., "Measurement of flow harmonics with multi-particle cumulants in Pb+Pb collisions at $\sqrt{S_{NN}} = 2.76$ TeV with the ATLAS detector," *The European Physical Journal C*, vol. 11, p. 3157, 2014.

[44] S. Chatrchyan, K. A. Bloom, B. Bockelman et al., "Measurement of higher-order harmonic azimuthal anisotropy in PbPb collisions at $\sqrt{S_{NN}} = 2.76$ TeV," *Physical Review C*, vol. 89, Article ID 044906, 2014.

[45] G. Aad, B. Abbott, J. Abdallah et al., "Measurement of the correlation between flow harmonics of different order in lead-lead collisions at $\sqrt{s_{NN}} = 2.76$ TeV with the ATLAS detector," *Physical Review C*, vol. 92, no. 3, Article ID 034903, 30 pages, 2015.

[46] S. Chatrchyan, V. Khachatryan, A. M. Sirunyan et al., "Centrality dependence of dihadron correlations and azimuthal anisotropy harmonics in PbPb collisions at $\sqrt{s_{NN}} = 2.76$ TeV," *The European Physical Journal C*, vol. 72, article 2012, 2012.

[47] S. Chatrchyan, D. Adamová, M. M. Aggarwal et al., "Azimuthal anisotropy of charged particles at high transverse momenta in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV," *Physical Review Letters*, vol. 109, no. 2, Article ID 022301, 2012.

[48] S. Chatrchyan, D. Tonoiu, P. Azzurri et al., "Measurement of the elliptic anisotropy of charged particles produced in PbPb collisions at $\sqrt{S_{NN}} = 2.76$ TeV," *Physical Review C*, vol. 87, no. 1, Article ID 014902, 2013.

[49] S. Chatrchyan, V. Khachatryan, A. M. Sirunyan et al., "Measurement of the azimuthal anisotropy of neutral pions in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ Tev," *Physical Review Letters*, vol. 110, no. 4, Article ID 042301, 15 pages, 2013.

[50] S. Chatrchyan, V. Khachatryan, A. M. Sirunyan et al., "Multiplicity and transverse momentum dependence of two- and four-particle correlations in pPb and PbPb collisions," *Physics Letters B*, vol. 724, no. 4-5, pp. 213–240, 2013.

[51] S. Chatrchyan, V. Khachatryan, A. M. Sirunyan et al., "Measurement of higher-order harmonic azimuthal anisotropy in PbPb collisions at $\sqrt{s_{NN}} = 2.76$ TeV," *Physical Review C*, vol. 89, no. 4, Article ID 044906, 2014.

[52] S. Chatrchyan, V. Khachatryan, A. M. Sirunyan et al., "Studies of azimuthal dihadron correlations in ultra-central Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ Tev," *Journal of High Energy Physics*, vol. 2014, no. 2, article 88, pp. 1–38, 2014.

[53] V. Khachatryan, A. M. Sirunyan, A. Tumasyan et al., "Long-range two-particle correlations of strange hadrons with charged particles in pPb and PbPb collisions at LHC energies," *Physics Letters B*, vol. 742, pp. 200–224, 2015.

[54] V. Khachatryan, A. M. Sirunyan, A. Tumasyan et al., "Evidence for collective multiparticle correlations in p − Pb collisions," *Physical Review Letters*, vol. 115, no. 1, Article ID 012301, 2015.

[55] V. Khachatryan, A. M. Sirunyan, A. Tumasyan et al., "Evidence for transverse-momentum- and pseudorapidity-dependent event-plane fluctuations in PbPb and pPb collisions," *Physical Review C*, vol. 92, no. 3, Article ID 034911, 2015.

[56] P. Huovinen, P. F. Kolb, U. W. Heinz, P. V. Ruuskanen, and S. A. Voloshin, "Radial and elliptic flow at RHIC: further predictions," *Physics Letters B*, vol. 503, no. 1-2, pp. 58–64, 2001.

[57] P. F. Kolb, P. Huovinen, U. Heinz, and H. Heiselberg, "Elliptic flow at SPS and RHIC: from kinetic transport to hydrodynamics," *Physics Letters B*, vol. 500, no. 3-4, pp. 232–240, 2001.

[58] M. Luzum and P. Romatschke, "Conformal relativistic viscous hydrodynamics: applications to RHIC results at $\sqrt{s_{NN}} = 200$ GeV ," *Physical Review C*, vol. 78, Article ID 034915, 2008.

[59] M. Luzum and P. Romatschke, "Viscous hydrodynamic predictions for nuclear collisions at the LHC," *Physical Review Letters*, vol. 103, no. 26, Article ID 262302, 2009.

[60] H. Song and U. W. Heinz, "Causal viscous hydrodynamics in $2+1$ dimensions for relativistic heavy-ion collisions," *Physical Review C*, vol. 77, no. 6, Article ID 064901, 2008.

[61] H. Song, S. A. Bass, U. Heinz, T. Hirano, and C. Shen, "200 A GeV Au + Au collisions serve a nearly perfect quark-gluon liquid," *Physical Review Letters*, vol. 106, no. 19, Article ID 192301, 2011.

[62] H. Niemi, K. J. Eskola, and R. Paatelainen, "Event-by-event fluctuations in a perturbative QCD + saturation + hydrodynamics model: Determining QCD matter shear viscosity in ultrarelativistic heavy-ion collisions," *Physical Review C*, vol. 93, no. 2, Article ID 024907, 2016.

[63] U. Heinz and R. Snellings, "Collective flow and viscosity in relativistic heavy-ion collisions," *Annual Review of Nuclear and Particle Science*, vol. 63, no. 1, pp. 123–151, 2013.

[64] M. Luzum and H. Petersen, "Initial state fluctuations and final state correlations in relativistic heavy-ion collisions," *Journal of Physics G: Nuclear and Particle Physics*, vol. 41, no. 6, Article ID 063102, 2014.

[65] P. Huovinen, "Hydrodynamics at RHIC and LHC: what have we learned?" *International Journal of Modern Physics E*, vol. 22, no. 12, Article ID 1330029, 2013.

[66] E. Shuryak, "Heavy ion collisions: achievements and challenges," https://arxiv.org/abs/1412.8393.

[67] H. Song, "Hydrodynamic modelling for relativistic heavy-ion collisions at RHIC and LHC," *Pramana*, vol. 84, no. 5, pp. 703–715, 2015.

[68] K. Dusling, W. Li, and B. Schenke, "Novel collective phenomena in high-energy proton-proton and proton-nucleus collisions," *International Journal of Modern Physics E*, vol. 25, no. 1, Article ID 1630002, 2016.

[69] P. Kovtun, D. T. Son, and A. O. Starinets, "Viscosity in strongly interacting quantum field theories from black hole physics," *Physical Review Letters*, vol. 94, no. 11, Article ID 111601, 2005.

[70] X. Zhu, Y. Zhou, H. Xu, and H. Song, "Correlations of flow harmonics in 2.76 A TeV Pb-Pb collisions," http://arxiv.org/abs/1608.05305.

[71] R. S. Bhalerao, J.-Y. Ollitrault, and S. Pal, "Characterizing flow fluctuations with moments," *Physics Letters B*, vol. 742, pp. 94–98, 2015.

[72] Y. Zhou, K. Xiao, Z. Feng, F. Liu, and R. Snellings, "Anisotropic distributions in a multiphase transport model," *Physical Review C*, vol. 93, no. 3, Article ID 034909, 2016.

[73] G. Giacalone, L. Yan, J. Noronha-Hostler, and J. Y. Ollitrault, "Symmetric cumulants and event-plane correlations in Pb + Pb collisions," *Physical Review C*, vol. 94, no. 1, Article ID 014906, 2016.

[74] R. Snellings, "Anisotropic flow at the LHC measured with the ALICE detector," *Journal of Physics G: Nuclear and Particle Physics*, vol. 38, no. 12, Article ID 124013, 2011.

[75] Y. Zhou, *Anisotropic flow and flow fluctuations at the large hadron collider [Ph.D. thesis]*, Utrecht University, CERN-THESIS-2016-005, 2016.

[76] L. Yan, J.-Y. Ollitrault, and A. M. Poskanzer, "Eccentricity distributions in nucleus-nucleus collisions," *Physical Review C*, vol. 90, no. 2, Article ID 024903, 9 pages, 2014.

[77] L. Yan and J. Ollitrault, "Universal fluctuation-driven eccentricities in proton-proton, proton-nucleus, and nucleus-nucleus collisions," *Physical Review Letters*, vol. 112, no. 8, Article ID 082301, 2014.

[78] L. Yan, J.-Y. Ollitrault, and A. M. Poskanzer, "Azimuthal anisotropy distributions in high-energy collisions," *Physics Letters B*, vol. 742, pp. 290–295, 2015.

[79] L. V. Bravina, E. S. Fotina, V. L. Korotkikh et al., "Anisotropic flow fluctuations in hydro-inspired freeze-out model for relativistic heavy ion collisions," *The European Physical Journal C*, vol. 75, no. 12, article 588, pp. 1–10, 2015.

[80] J. Jia, "Event-shape fluctuations and flow correlations in ultra-relativistic heavy-ion collisions," *Journal of Physics G: Nuclear and Particle Physics*, vol. 41, no. 12, Article ID 124003, 2014.

[81] H. Petersen, G. Y. Qin, S. A. Bass, and B. Müller, "Triangular flow in event-by-event ideal hydrodynamics in Au + Au collisions at $\sqrt{s_{NN}} = 200A$ GeV," *Physical Review C*, vol. 82, no. 4, Article ID 041901, 2010.

[82] Z. Qiu and U. W. Heinz, "Event-by-event shape and flow fluctuations of relativistic heavy-ion collision fireballs," *Physical Review C*, vol. 84, no. 2, Article ID 024911, 2011.

[83] H. Niemi, G. S. Denicol, H. Holopainen, and P. Huovinen, "Event-by-event distributions of azimuthal asymmetries in ultrarelativistic heavy-ion collisions," *Physical Review C*, vol. 87, no. 5, Article ID 054901, 2013.

[84] U. Heinz, Z. Qiu, and C. Shen, "Fluctuating flow angles and anisotropic flow measurements," *Physical Review C*, vol. 87, no. 3, Article ID 034913, 2013.

[85] F. G. Gardim, F. Grassi, M. Luzum, and J.-Y. Ollitrault, "Breaking of factorization of two-particle correlations in hydrodynamics," *Physical Review C—Nuclear Physics*, vol. 87, no. 3, Article ID 031901, 2013.

[86] Y. Zhou, "Searches for pT dependent fluctuations of flow angle and magnitude in Pb-Pb and p-Pb collisions," *Nuclear Physics A*, vol. 931, pp. 949–953, 2014.

[87] I. Kozlov, M. Luzum, G. S. Denicol, S. Jeon, and C. Gale, "Signatures of collective behavior in small systems," *Nuclear Physics A*, vol. 931, pp. 1045–1050, 2014.

[88] A. Andronic, G. Stoicea, M. Petrovici et al., "Transition from in-plane to out-of-plane azimuthal enhancement in Au + Au collisions," *Nuclear Physics A*, vol. 679, no. 3-4, pp. 765–792, 2001.

[89] P. Chung, N. N. Ajitanand, J. M. Alexander et al., "Centrality and momentum-selected elliptic flow: tighter constraints for the nuclear equation of state," *Physical Review C*, vol. 66, no. 2, Article ID 021901, 5 pages, 2002.

[90] J. Adams, C. Adler, M. M. Aggarwal et al., "Azimuthal anisotropy at the relativistic heavy ion collider: the first and fourth harmonics," *Physical Review Letters*, vol. 92, no. 6, Article ID 062301, 6 pages, 2004.

[91] D. Teaney and L. Yan, "Event-plane correlations and hydrodynamic simulations of heavy ion collisions," *Physical Review C*, vol. 90, no. 2, Article ID 024902, 2014.

[92] R. S. Bhalerao, J. Y. Ollitrault, and S. Pal, "Event-plane correlators," *Physical Review C*, vol. 88, no. 2, Article ID 024909, 2013.

[93] A. Bilandzic, C. H. Christensen, K. Gulbrandsen, A. Hansen, and Y. Zhou, "Generic framework for anisotropic flow analyses with multiparticle azimuthal correlations," *Physical Review C*, vol. 89, no. 6, Article ID 064904, 2014.

[94] R. S. Bhalerao, M. Luzum, and J.-Y. Ollitrault, "Understanding anisotropy generated by fluctuations in heavy-ion collisions," *Physical Review C*, vol. 84, no. 5, Article ID 054901, 8 pages, 2011.

[95] D. Teaney and L. Yan, "Nonlinearities in the harmonic spectrum of heavy ion collisions with ideal and viscous hydrodynamics," *Physical Review C*, vol. 86, no. 4, Article ID 044908, 2012.

[96] J. Schukraft, A. Timmins, and S. A. Voloshin, "Ultra-relativistic nuclear collisions: event shape engineering," *Physics Letters B*, vol. 719, no. 4-5, pp. 394–398, 2013.

[97] P. Huo, J. Jia, and S. Mohapatra, "Elucidating the event-by-event flow fluctuations in heavy-ion collisions via the event-shape selection technique," *Physical Review C*, vol. 90, no. 2, Article ID 024910, 2014.

[98] X.-N. Wang and M. Gyulassy, "Hijing: a Monte Carlo model for multiple jet production in pp, pA, and AA collisions," *Physical Review D*, vol. 44, no. 11, pp. 3501–3516, 1991.

[99] M. Gyulassy and X.-N. Wang, "HIJING 1.0: a Monte Carlo program for parton and particle production in high energy hadronic and nuclear collisions," *Computer Physics Communications*, vol. 83, no. 2-3, pp. 307–331, 1994.

[100] Y. Zhou, S. S. Shi, K. Xiao, K. J. Wu, and F. Liu, "Higher moments of net baryon distribution as probes of the QCD critical point," *Physical Review C*, vol. 82, Article ID 014905, 2010.

[101] Z.-W. Lin, C. M. Ko, B.-A. Li, B. Zhang, and S. Pal, "Multiphase transport model for relativistic heavy ion collisions," *Physical Review C*, vol. 72, no. 6, Article ID 064901, 29 pages, 2005.

[102] Y. Zhou, "Measurements of correlations of anisotropic flow harmonics in Pb-Pb Collisions with ALICE," https://arxiv.org/abs/1512.05397.